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DRAINAGE AND EVAPORATION OF A LIQUID FILM WITH SALT DEPOSITION UPON A HORIZONTAL CYLINDRICAL SURFACE

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Abstract—An analysis is made of the drainage and evaporation of a liquid film upon a horizontal cylindrical surface which forms the interior or exterior surface of a tube. The results are expressed in Fig. 2 in terms of time to the first appearance to dryout versus the film thickness, with a Reynolds numbers as a parameter.

The analysis of the drainage and evaporation process is used as a starting point for the determination of the salt deposited at dryout and the major results are expressed in Fig. 5. It is noteworthy that it appears that there is no substantial dependence of the amount of salt deposited upon the molecular diffusivity.

NOMENCLATURE

a_1, a_2,	coefficients in equation (3.1);
b_1, b_2,	coefficients in equation (7.4);
c,	concentration;
F,	defined by equation (2.5);
g,	gravitational constant
h,	film thickness;
H_v, H_s,	dimensionless film thicknesses. See equations (3.7) and (6.4);
L_v, L_s,	characteristic lengths. See equations (3.8) and (6.5);
m,	mass of salt per unit area of tube wall;
m_1,	component of m defined by equation (9.1);
M_v, M_s,	dimensionless salt masses. See equations (8.7) and (8.11). An additional subscript 1 denotes values based upon m_1;
p,	pressure;
Q,	heat flux;
r,	tube radius;
R,	Reynolds number defined by equation (4.7);
t,	time;
u,	x-component of velocity;
u_f, u_F,	value of u at free surface and for free fall respectively;
$\bar{u}$,	average value of u defined by equation (2.12);
v,	y-component of velocity;
v_h,	value of v at $y = h$;
w,	velocity due to evaporation defined by equation (2.13);
x,	coordinate parallel to wall from top of perimeter;
X,	body force component in Navier–Stokes equation. See equation (2.3);
y,	coordinate normal to and from the wall;
Y,	body force component in Navier–Stokes equation. See equation (2.4);
z,	coordinate normal to free surface into liquid.

Greek letters

η,	dimensionless distance, equation (4.4);
η_h,	value of η at $y = h$;
θ_v, θ_s,	dimensionless time defined by equations (3.7) and (6.4);
λ,	latent heat of vaporization;

μ,	liquid viscosity;
ν,	kinematic viscosity;
ρ,	liquid density;
ρ_G,	light phase density;
$\Delta\rho$,	density difference between phases;
ϕ,	dimensionless velocity, equation (4.3);
ϕ_h,	value of ϕ at $y = h$.

Subscripts O and D denote conditions for original film and at dryout.

1. INTRODUCTION

THE WORK to be presented consists of an analysis of the drainage and evaporation of a liquid film upon a uniformly heated horizontal cylindrical surface, which may either be the interior or exterior surface of a tube. Attention will chiefly be focussed upon the time it takes for the surface to become dry and upon the amount of involatile but soluble salt that is then left behind near the top of the tube's perimeter.

The sequence of events to be considered is fairly complex and the following qualitative description is helpful before the mathematical analysis is attempted. A uniform layer of liquid is initially laid down upon the surface and it will be shown that this layer subsequently remains substantially uniform as it thins near the top of that surface, which is the region to which attention is devoted because it is there that dryout first occurs. The majority of the liquid rapidly attains the 'free-fall' velocity from a value of zero at the point of symmetry at the top of the perimeter and a stable, time-independent boundary layer builds up against the tube wall. It will be shown that this boundary layer is identical with that for a two-dimensional stagnation point since the free-fall velocity which exists outside of it, varies linearly with distance along the surface. The known solution for this situation will be allied with a volumetric balance for the film, which incorporates a loss of liquid due to evaporation, which is assumed to occur uniformly over the whole liquid–vapour interface.

The solution just described may sufficiently account for the whole drainage and evaporation process. The rate of film thinning due to drainage will decrease as the film thins and the rate due to evaporation will become dominant. If the latter happens before the vapour–liquid interface enters the boundary layer, when the conditions necessary for the two-dimensional stagnation point solution cease to exist, then no further complications need to be considered. In many cases, however, the heat flux will be insufficient for this situation to obtain and another solution is developed, in which fluid inertia is neglected in the drainage process, so that the velocity profile within the film is the usual parabolic one associated with purely viscous flow. The two solutions described will be patched to form an adequate description of the whole drainage and evaporation process.

A study related to the present one is that of Butterworth [1] who examined the purely viscous drainage of a liquid upon the surface of a rod, which was slightly inclined to the vertical. He estimated the variation of film thickness around the whole perimeter of the rod and his graphs clearly show that the assumption that the film thickness does not vary with position near the top of the perimeter should provide a useful approximation. The variation of film thickness will be estimated, however, in Section 7 of the present analysis in order to determine the rate of expansion of a dry region, once formed, as well as to estimate the distance from the top of the tube, for which the analysis should be adequate.

The estimation of the amount of salt left behind at dryout requires the knowledge obtained of the flow fields in the drainage process. The analysis is simplified by the fact that, as long as the initial concentration is uniform, there will be no concentration variations parallel to the tube wall. Two extreme cases are therefore examined. In one the diffusivity of the salt is assumed to be so large that the concentration is also invarient with depth in the liquid film. In the other case the diffusivity is assumed to be so small that the variations in concentration are limited to liquid layers immediately adjacent to the vapour–liquid interface. Surprisingly, the choice of assumption is only found to make, at most, less than 7 per cent difference to the result.

The results of the present work are chiefly of interest to the writer with respect to boiling water in horizontal tubes at pressures between 6 and 18 MN/m^2. Dryout of the tube wall might in some cases elevate the wall temperature sufficiently to cause ductile failure of the tube. Alternatively the thin internal layer of magnetite, which protects the carbon steel from further oxidation, may be cracked by the thermal stresses. Lastly and perhaps most importantly, deposited salts may corrode the tube and the consequent thinning of the wall may also lead to ductile failure. Now experience with two-phase flow regimes leads one to expect that, under certain circumstances, disturbances may periodically pass down a tube and wet the walls. A simple example occurs in slug flow, where slugs of liquid separate large, long bubbles, with only a thin layer of liquid against the upper wall of the tube. It is such a liquid layer that is of concern. A more complex situation is that where the slugs can be regarded as having degenerated into large waves which provide a 'wetting event', depositing a liquid film that is then the subject of drainage and evaporation.

A practical example which agrees with the above description is afforded by Lis and Strickland [4], who boiled water in a system of horizontal tubes of 41 mm bore connected by upward return bends. The pressure was 6·6 MN/m^2 and the heat flux 0·06 MW/m^2. Under certain flow conditions, where the proportion of steam was large, the upper wall of the tube was observed to have temperature fluctuations with a period of between 2 and 5 s. The theory to be given in the present paper would predict a time to dryout of 0·8 s for the same conditions. It is of interest to note that, for this example, the film thickness is only of importance to the calculation when it falls to about 70 μm. Any surplus water drains too rapidly to affect the calculated time.

2. REDUCTION OF THE NAVIER–STOKES EQUATIONS

The system is illustrated in Fig. 1. A film of thickness h is shown on the inside surface of a tube of radius r. x is measured parallel to the wall from the radius intersecting the top of the perimeter and y is measured from the wall towards the axis. We only consider $y \leqslant h$ and (h/r) is sufficiently small for the usual boundary-layer approximation to be accepted that x and y are rectangular coordinates, with u and v being the respective velocity components.

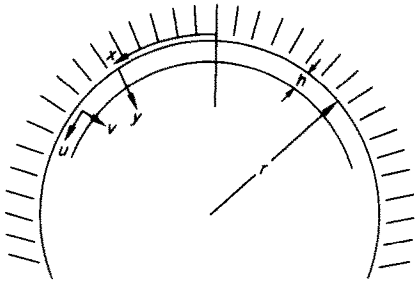

FIG. 1. Coordinate system.

The Navier–Stokes equations are

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right] + X \qquad (2.1)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right] + Y \qquad (2.2)$$

where t is time, ρ is the liquid density, ν is the liquid kinematic viscosity and p is pressure. The body force components X and Y are

$$X = g\sin\left(\frac{x}{r}\right) \simeq g\frac{x}{r} \qquad (2.3)$$

$$Y = g\cos\left(\frac{x}{r}\right) \simeq g \qquad (2.4)$$

where the approximations are accepted, except in Section 7, for the remainder of this paper.

Now let us determine the consequences of the assumption

$$v = -F(y, t). \qquad (2.5)$$

The continuity equation for an incompressible medium is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \qquad (2.6)$$

With the boundary condition that $u = v = 0$ when $x = 0$, we therefore find that

$$u = x\frac{\partial F}{\partial y}. \qquad (2.7)$$

Substitution of equations (2.5) and (2.7) in equation (2.2) results in an equation for $\partial p/\partial y$ which is independent of x. It follows that $\partial p/\partial x$ is independent of y and will have the value in the vapour phase immediately adjacent to the liquid–vapour interface. It is noted that, to the level of approximation being considered, h is independent of x and thus surface tension will have no contribution to make to $\partial p/\partial x$ in the liquid, owing to any gradient of the curvature of the interface with respect to x. Set p arbitrarily equal to zero at the vapour–liquid interface at $x = 0$. We have at other positions along the interface

$$p = \rho_G g r\left[1 - \cos\left(\frac{x}{r}\right)\right] \simeq \rho_G g\frac{x^2}{2r} \qquad (2.8)$$

$$\frac{\partial p}{\partial x} \simeq \rho_G g\frac{x}{r} \qquad (2.9)$$

where ρ_G is the vapour phase density. Equation (2.1) now becomes

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + \left(\frac{\Delta\rho g}{\rho r}\right)x \qquad (2.10)$$

where $\Delta\rho$ is the density difference between the phases and it is noted that $\partial^2 u/\partial x^2$ has been set equal to zero in agreement with equation (2.7).

Before equation (2.10) can be accepted as the required reduction of the Navier–Stokes equations, it must be shown that equation (2.7) satisfies the condition that the film thickness, h, only varies with time, since that assumption was implicit in the reduction process. The volumetric balance for the film is

$$\frac{\partial h}{\partial t} + \frac{\partial(h\bar{u})}{\partial x} + w = 0 \qquad (2.11)$$

where $\bar{u}$ is an average velocity defined by

$$h\bar{u} = \int_0^h u\,\mathrm{d}y \qquad (2.12)$$

and w is the velocity of film thinning owing to evaporation

$$w = \left(\frac{Q}{\rho\lambda}\right) \qquad (2.13)$$

where Q is the uniform heat flux and λ is the latent heat of vaporization. Note that w has a direction opposite to that of v and also that, although it can be a function of t, it is taken to be constant throughout the rest of this paper.

Now if h is assumed for the moment to be invarient in x, we can apply equations (2.7) and (2.12) to equation (2.11) and obtain an equation for $\partial h/\partial t$ which is independent of x. Thus, as long as h is originally independent of x, it will remain so and equation (2.10) can be accepted. The physical reasons for this conclusion may be made clearer by the following more qualitative description. Regard a film of uniform thickness which is also thinning uniformly. Part of the rate of thinning is accounted for by w in equation (2.11) which leaves a uniform rate which must be associated with flow within the film. The average film velocity, $\bar{u}$, consequently increases linearly with x from a zero value at $x = 0$. This statement is clearly compatible with equation (2.7) for local values of velocity u and, through the continuity equation, with equation (2.5) for the local velocity v. We must, however, satisfy ourselves that the velocities in question can be obtained with the applied gravitational force. It is found that substitution of equations (2.5) and (2.7) in the reduced Navier–Stokes equation (2.10) yields an equation for F in terms of the independent variables y and t only and thus the assumption that F is not a function of x is justified. It is noted that equation (2.10) actually describes a set of motive and resistive forces which are all linear in x and this provides the main physical reason why the uniform film thinning is matched by the forces available. The reason, of course, relies strongly upon the approximation of equation (2.3) which is dependent upon the shape of the surface. Nonetheless, it may be noted that a surface shape can be defined for which the approximation of equation (2.3) is unnecessary.

The various approximations required to obtain easily managed equations may appear unduly restrictive but they will be examined more closely in Section 7, where the rôle of surface tension will also be reassessed.

3. EVAPORATION WITH PURELY VISCOUS DRAINAGE

Emphasis was placed in the introduction upon starting the sequence of events with a thick liquid film. From the analytical point of view, however, dryout is the more convenient origin for the time coordinate and, as we proceed into negative time, we first of all encounter a purely viscous drainage process.

Replace equation (2.7) with

$$\frac{u}{x} = a_1 y + a_2 y^2 \tag{3.1}$$

where a_1 and a_2 are functions of time only. Using the continuity equation (2.6)

$$v = -\frac{a_1 y^2}{2} - \frac{a_2 y^3}{3}. \tag{3.2}$$

Substituting equations (3.1) and (3.2) and their derivatives in equation (2.10), we obtain an equation independent of x. By placing the sum of terms independent of y equal to zero we find

$$a_2 = -\left[\frac{\Delta\rho g}{2\mu r}\right] \tag{3.3}$$

where μ is the liquid viscosity. It is noted that the inertia terms of equation (2.10) have not been used.

The coefficient a_1 is determined by employing the boundary condition that there is no shear on the vapour–liquid interface or

$$\left(\frac{\partial u}{\partial y}\right)_{y=h} = 0. \tag{3.4}$$

Thus

$$a_1 = -2a_2 h. \tag{3.5}$$

The value of $\bar{u}$ can now be estimated from equation (2.12) and substitution in equation (2.11), followed by rearrangement, yields

$$\frac{\mathrm{d}H_v}{\mathrm{d}\theta_v} + H_v^3 + 1 = 0 \tag{3.6}$$

where the following dimensionless quantities are defined

$$H_v = \left(\frac{h}{L_v}\right), \theta_v = \left(\frac{wt}{L_v}\right) \tag{3.7}$$

where the characteristic length is

$$L_v = \left[\frac{3\mu r w}{\Delta\rho g}\right]^{\frac{1}{3}}. \tag{3.8}$$

Integration of equation (3.6), with the boundary condition that $H_v = 0$ when $\theta_v = 0$, yields

$$-\theta_v = \frac{1}{6}\ln\left[\frac{(1+H_v)^3}{1+H_v^3}\right] + \frac{1}{3^{\frac{1}{2}}}\left[\tan^{-1}\left[\frac{2H_v - 1}{3^{\frac{1}{2}}}\right] + \frac{\pi}{6}\right]. \tag{3.9}$$

Equation (3.9) is drawn in Fig. 2 and it is seen that θ_v tends to a constant value as H_v increases. In fact

$$(-\theta_v)_{H_v \to \infty} = \frac{2\pi}{3^{\frac{3}{2}}}. \tag{3.10}$$

Lastly the condition of validity for assuming that the acceleration and momentum terms in equation (2.10) are negligible will be determined. Equations (3.1) and (3.2) and their derivatives are substituted in equation (2.10), with the values of a_1 and a_2 given by equations (3.3) and (3.5). Equation (3.6) is employed to define $(\mathrm{d}H_v/\mathrm{d}\theta_v)$ and it is found that the terms in question are negligible if $H_v\ (H_v^3 + 1)\ R \ll 1$, where R is a Reynolds number given by

$$R = \left(\frac{wL_v}{\nu}\right). \tag{3.11}$$

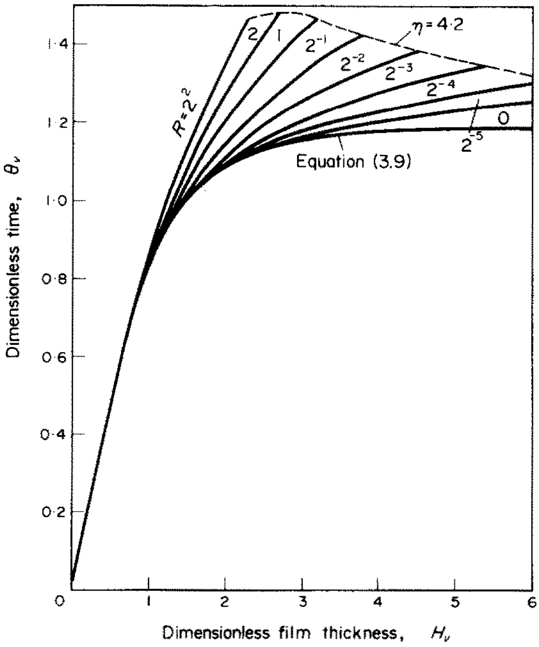

FIG. 2. Time to dryout.

4. EVAPORATION WITH STAGNATION FLOW

The results of the last section imply that the liquid velocity at the vapour–liquid interface increases indefinitely with film thickness. Such a conclusion is clearly ridiculous and in this section we are to seek a solution such that liquid velocity at the interface is restricted to that of free-fall from the topmost position upon the cylindrical surface. The liquid at the interface will thus satisfy the energy requirements of a system without any energy dissipation. The free-fall velocity is

$$u_F = \left\{2gr\frac{\Delta\rho}{\rho}\left[1 - \cos\left(\frac{x}{r}\right)\right]\right\}^{\frac{1}{2}} \simeq \left(\frac{\Delta\rho g}{\rho r}\right)^{\frac{1}{2}} x \tag{4.1}$$

and has the property of two-dimensional stagnation flow, that the mainstream velocity varies linearly with distance from the stagnation point. Schlichting [5] presents and discusses the solutions provided by Hiemenz [2] and Howarth [3]. The solution sought is time-independent and equation (2.10) reduces to

$$\phi''' + \phi\phi'' - \phi'^2 + 1 = 0 \tag{4.2}$$

where ϕ is a function of η and primes denote differentiation with respect to η

$$\phi = -\left(\frac{\rho r}{\nu\Delta\rho g}\right)^{\frac{1}{2}} v \tag{4.3}$$

$$\eta = \left(\frac{\Delta\rho g}{\rho r}\right)^{\frac{1}{2}} y. \tag{4.4}$$

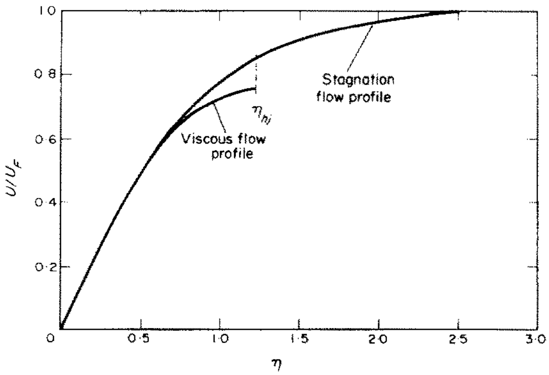

FIG. 3. Comparison of stagnation flow with viscous flow at patching of flow regimes.

Figure 3 illustrates the u-component of velocity derived after integration of equation (4.2). It is clear that, as long as the liquid–vapour interface is situated at a sufficiently large value of η, which will be designated η_h, the stagnation flow solution for $\eta \leqslant \eta_h$ will adequately represent conditions within the thinning film. We can determine the v-component of velocity at the interface, v_h, from ϕ_h which exists at $\eta = \eta_h$ and use the tabulation of ϕ versus η given by Schlichting [5].

The volumetric balance for the thinning film takes the form

$$\frac{\mathrm{d}h}{\mathrm{d}t} = v_h - w \qquad (4.5)$$

which is readily reduced to

$$\frac{\mathrm{d}H_v}{\mathrm{d}\theta_v} = -\frac{3^{\frac{1}{4}}}{R^{\frac{3}{4}}}\phi_h - 1. \qquad (4.6)$$

Equation (4.6) can be integrated numerically, using the table of ϕ versus η, if η_h is reduced to the form

$$\eta_h = (3R)^{\frac{1}{4}} H_v. \qquad (4.7)$$

The integration is valid for sufficiently large values of η_h.

5. PATCHING THE VISCOUS DRAINAGE AND STAGNATION FLOW SOLUTIONS

The viscous drainage and stagnation flow solutions are to be patched at the film thickness for which the estimated wall shear stress or, in other words, the velocity gradient at $y = 0$ is the same for both solutions. Other criteria for patching could be chosen, such for example that the total estimated film flow rate is the same for both solutions, but the chosen criterion is appropriate.

It is readily deduced from equations (3.1), (3.3) and (3.5) that for the viscous flow solution

$$\frac{1}{x}\left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{\Delta\rho g h}{\mu r}. \qquad (5.1)$$

We can find the corresponding value of the velocity gradient for stagnation flow by observing from equations (2.5) and (2.6) that

$$\frac{1}{x}\frac{\partial u}{\partial y} = -\frac{\partial^2 v}{\partial y^2} \qquad (5.2)$$

and obtaining from Schlichting's table

$$\phi''_{\eta=0} = 1{\cdot}2326. \qquad (5.3)$$

Thus the patching criterion reduces to definition of either a value of H_{vj} or of η_{hj} for the patch as follows:

$$H_{vj} = \frac{1{\cdot}2326}{(3R)^{\frac{1}{4}}}, \eta_{hj} = 1{\cdot}2326. \qquad (5.4)$$

The viscous velocity profile at the patch is given by

$$\frac{u}{u_F} = 1{\cdot}2326\eta - \tfrac{1}{2}\eta^2 \qquad (5.5)$$

which is drawn in Fig. 3 for comparison with the stagnation flow profile. It is seen that the assumed transition from one flow regime to another is not very abrupt, at least with respect to the important variable $\bar{u}$, which is used in the volumetric balance.

6. DISCUSSION OF THE ESTIMATED TIME TO DRYOUT

All the appropriate calculation indicated in the last two sections have been made and are summarized in the time-to-dryout curves of Fig. 2, where $(-\theta_v)$ is shown as a function of H_v with R as a parameter.

The viscous flow solution appears to provide a useful approximation when R is less than about 0·05 and then the result of equation (3.10) is particularly valuable because it gives a time to dryout which is independent of the original film thickness, as long as that thickness is large enough. It must be noted, however, that very thick films require the stagnation flow analysis and thus there are some reservations to the conclusion just stated.

If the value of R is sufficiently large, the estimated time to dryout becomes almost independent of the viscous flow solution. The reason for this is that the rate of film thinning is dependent upon both evaporation and drainage and the evaporation becomes dominant when the film is sufficiently thin. In this respect it is noted that both equation (3.6) for the viscous flow solution and equation (4.6) for the stagnation flow solution show that $\mathrm{d}H_v/\mathrm{d}\theta_v$ tends to (-1) as H_v tends to zero. Now, if the evaporation rate is so large that it becomes dominant before η_h reduces to 1·2326, then the viscous flow solution is unimportant.

Let us consider situations where the evaporation rate is dominant before η_h reduces to 2·5, when Fig. 3 suggests that a simple solution might be obtained. In that event ϕ is linear in η

$$\phi = \eta - 0{\cdot}6482 \tag{6.1}$$

and equation (4.6) becomes

$$\frac{\mathrm{d}H_v}{\mathrm{d}\theta_v} = -\left(\frac{3}{R}\right)^{\frac{1}{2}} H_v - 1 + 0{\cdot}6482\left(\frac{3}{R^3}\right)^{\frac{1}{4}}. \tag{6.2}$$

Integration of equation (6.2) with the boundary condition that $H_v = 0$ when $\theta_v = 0$ leads to

$$-\theta_S = \ln\left[H_S + 1\right] \tag{6.3}$$

where

$$H_S = \left(\frac{h}{L_S}\right), \qquad \theta_S = \left(\frac{wt}{L_S}\right) \tag{6.4}$$

$$L_S = \left[\frac{\rho r w^2}{\Delta\rho g}\right]^{\frac{1}{2}} = \left(\frac{R}{3}\right)^{\frac{1}{2}} L_v. \tag{6.5}$$

Equation (6.3) indicates no limiting value of θ_S similar to that found for θ_v in viscous flow. The logarithmic form of the equations suggests, however, that a characteristic dryout time may be selected as being meaningful. We can write

$$\frac{\mathrm{d}H_S}{\mathrm{d}\theta_S} = -\exp(-\theta_S) \tag{6.6}$$

and arbitrarily choose the characteristic time θ_{SC} to be that at which the gradient expressed by equation (6.6) is ten times its value at $\theta_S = 0$. Thus $\theta_{SC} = -2{\cdot}3$ is chosen. It is noted that θ_S is independent of w or the heat flux, while θ_v of equations (3.7) vary as $w^{\frac{2}{3}}$.

7. VARIATION OF FILM THICKNESS WITH DISTANCE FROM THE TOP OF THE TUBE

It has been shown that, as long as the angle (x/r) is small, the variation of the film thickness with respect to x can be neglected for many purposes. The variation will not be negligible, however, if we require to estimate the rate of enlargement of a dry area and therefore expressions for the variation will be derived for the extreme cases of $R \ll 1$ and $R \gg 1$.

First consider a purely viscous drainage process. Assume, in place of equations (3.1) and (3.2), that

$$u = r\sin\left(\frac{x}{r}\right)\left[a_1 y + a_2 y^2\right] \tag{7.1}$$

$$v = -\cos\left(\frac{x}{r}\right)\left[\frac{a_1 y^2}{2} + \frac{a_2 y^3}{3}\right]. \tag{7.2}$$

We find that equations (7.1) and (7.2) provide a solution to the Navier–Stokes equations (2.1) and (2.2) with the momentum term omitted and without the approximations given in (2.3), (2.4) and (2.8) being applied. By performing the operations used to achieve equation (3.6) we arrive at

$$\frac{\partial H_v}{\partial\theta_v} + \frac{\partial\left[H_v^3 \sin(x/r)\right]}{\partial(x/r)} + 1 = 0 \tag{7.3}$$

instead.

A series form of solution is assumed as follows

$$H_v = H_{vT} + b_1(x/r)^2 + b_2(x/r)^4 + b_3(x/r)^6 + \ldots \tag{7.4}$$

and we also write

$$\frac{\partial H_v}{\partial\theta} = \frac{\partial H_v}{\partial H_{vT}}\frac{\mathrm{d}H_{vT}}{\mathrm{d}\theta}. \tag{7.5}$$

Equations (7.4) and (7.5) are substituted in (7.3) and sin (x/r) is expanded. The coefficients on $(x/r)^0$, $(x/r)^2$ and so on are all placed equal to zero to obtain a set of ordinary differential equations

$$\frac{\mathrm{d}H_{vT}}{\mathrm{d}\theta} = -\left[H_{vT}^3 + 1\right] \tag{7.6}$$

$$\frac{db_1}{dH_{vT}}[H_{vT}^3 + 1] = H_{vT}^2\left[9b_1 - \frac{H_{vT}}{2}\right] \quad (7.7)$$

and so on, which can be solved with appropriate boundary conditions. It is noted that equation (7.6) is identical with equation (3.6) and thus H_{vT} is the dimensionless film thickness at the top of the perimeter.

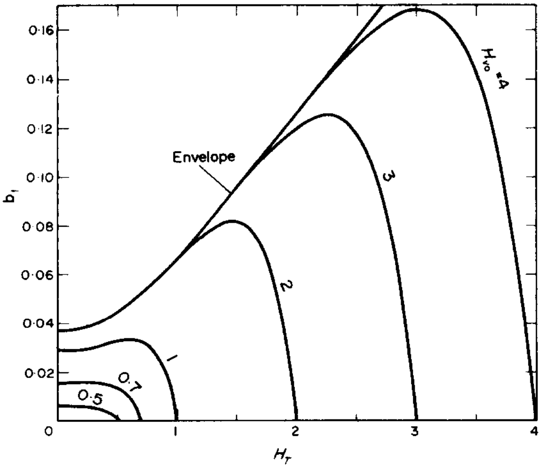

FIG. 4. Coefficient b_1.

$$H_v = 0{\cdot}0373(x/r)^2 + 0{\cdot}00186(x/r)^4 + 0{\cdot}000120(x/r)^6 + \ldots \quad (7.8)$$

and when $H_{vT} > 1{\cdot}5$

$$\frac{H_v}{H_{vT}} = 1 + 0{\cdot}0625(x/r)^2 + 0{\cdot}00400(x/r)^4 + 0{\cdot}000295(x/r)^6 + \ldots \quad (7.9)$$

Clearly H_v does not vary rapidly with x.

At this point it is worth pausing to consider the possible importance of the omission of two terms on the right of the Navier–Stokes equation (2.10), which should be included if the film thickness varies with x. The first is $[(\Delta\rho g/\rho)(\partial h/\partial x)\cos(x/r)]$ and accounts for the gradient of pressure with respect to x owing to the gradient of the film thickness. The second is $[(\sigma/\rho)\,\partial^3 h/\partial x^3]$, where σ is surface tension, and accounts for the change in the gradient of pressure in the film due to the gradient of the curvature of the interface with respect to x and surface tension. It is assumed that the curvature is adequately approximated by $(-\partial^2 h/\partial x^2)$. The consequence is that the second term of equation (7.3) should read

$$\frac{\partial\left\{H_v^3 \sin\left(\frac{x}{r}\right)\left[1 + \cot\left(\frac{x}{r}\right)\frac{\partial h}{\partial x} + \Sigma\,\mathrm{cosec}\left(\frac{x}{r}\right) r^2 \frac{\partial^3 h}{\partial x^3}\right]\right\}}{\partial\left(\frac{x}{r}\right)} \quad (7.10)$$

The solution for b_1 with the boundary condition that $b_1 = 0$ when H_{vT} has its initial value, is given in Fig. 4. It is noted that, as long as this initial value is greater than about 1·5, the curve for b_1 closely approaches an envelope curve before the occurrence of dryout. This envelope has two useful properties which are also found for b_2, b_3, etc. They are that b has a non-zero positive value at $H_{vT} = 0$, when dryout occurs and that b is linear in H_{vT} for H_{vT} greater than about 1·5. Thus, at dryout, we find we can write for films which were initially sufficiently thick

where

$$\Sigma = \left[\frac{\sigma}{\Delta\rho g r^2}\right]. \quad (7.11)$$

The two new terms are obvious in (7.10). We substitute the appropriate differentials from equation (7.9) into (7.10) and find that the two terms can be neglected if $(h_T/8r)$ and 0·096 $\Sigma\,(h_T/r)$ respectively are much less than unity. It is a condition of the analysis that (h/r) is much less than unity so that it only remains to show that $\Sigma < 1$. The last condition obtains for the air–water system at room temperature if $r > 3$ mm, while it obtains for the steam–water system at 18 MN/m^2 if $r > 0{\cdot}8$ mm. Thus the additional terms may be neglected in many practical instances. It is to be noted that equation (7.8) does not have to be considered in this respect since the rate of film thinning at dryout is chiefly by evaporation.

Now let us consider the situation when $R \gg 1$ so that free fall velocity of equation (4.1) can be assumed to obtain throughout the liquid film. Substitution in equation (2.11) yields

$$\frac{\partial H_S}{\partial \theta_S} + 2^{\frac{1}{2}}\frac{\partial\{H_S[1 - \cos(x/r)]^{\frac{1}{2}}\}}{\partial(x/r)} + 1 = 0. \tag{7.12}$$

It is found that a closely similar development to that following equation (7.3) is possible, with the result that, if the original film is thick enough, we can write at dryout

$$H_S = \frac{1}{48}\left(\frac{x}{r}\right)^2 + \frac{1}{720}\left(\frac{x}{r}\right)^4 + 0{\cdot}0000602\left(\frac{x}{r}\right)^6 + \ldots \tag{7.13}$$

and for H_{ST} larger than 1·5

$$\frac{H_S}{H_{ST}} = 1 + \frac{1}{16}\left(\frac{x}{r}\right)^2 + \frac{1}{384}\left(\frac{x}{r}\right)^4 + 0{\cdot}0000922\left(\frac{x}{r}\right)^6 + \ldots \tag{7.14}$$

It is clear from Fig. 2 that, as dryout is approached, the rate of film thinning is dominated by evaporation and it is therefore a simple matter to estimate the rate of expansion of the dry area from either equation (7.8) or (7.13), whichever is applicable.

8. DEPOSITION OF SALT WITH A LARGE DIFFUSIVITY

The analysis to be presented is limited to small values of the concentration, c. This restriction is likely to be violated as dryout is approached but nonetheless the estimate of the amount of salt deposited at dryout will not be substantially affected as long as the salt concentration in the original liquid is small.

Consider the migration of salt parallel to the tube wall. The salt balance for the whole film depth is

$$\frac{\partial\left[\int_0^h c\,\mathrm{d}y\right]}{\partial t} + \frac{\left[\partial \int_0^h cu\,\mathrm{d}y\right]}{\partial x} = 0. \tag{8.1}$$

Let us use the approximation that h is independent of x, when the form of equation (2.7) is suitable for u. Thus

$$\frac{\partial\left[\int_0^h c\,\mathrm{d}y\right]}{\partial t} + \int_0^h \frac{\mathrm{d}F}{\partial y}c\,\mathrm{d}y = 0 \tag{8.2}$$

if c is assumed independent of x. Equation (8.2) is independent of x and the assumption with respect to c will be valid if c is initially independent of x.

Now consider a salt with such a large diffusivity that the concentration is substantially uniform throughout the liquid depth and thus throughout the whole liquid film. Equation (8.1) reduces to

$$\frac{\partial(ch)}{\partial t} + ch\frac{\partial\bar{u}}{\partial x} = 0. \tag{8.3}$$

Equation (8.3) together with the volumetric balance of equation (2.11) yields

$$h\frac{\mathrm{d}c}{\mathrm{d}t} = cw. \tag{8.4}$$

Consider purely viscous drainage first. Equation (8.4) is rearranged to

$$H_v\frac{\mathrm{d}H_v}{\mathrm{d}\theta_v}\frac{\mathrm{d}c}{\mathrm{d}H_v} = c. \tag{8.5}$$

Substitute for $\mathrm{d}H_v/\mathrm{d}\theta_v$ from equation (3.6) and integrate

$$\frac{c}{c_0} = \frac{H_{v0}}{H_v}\left[\frac{1 + H_v^3}{1 + H_{v0}^3}\right]^{\frac{1}{3}} \tag{8.6}$$

where $c = c_0$ when $H_v = H_{v0}$.

Now the total mass of salt in the film is

$m = ch$ and a dimensionless mass of salt is defined as

$$M_v = \frac{c}{c_0} H_v = \frac{m}{c_0 L_v}. \qquad (8.7)$$

It is readily deduced from equation (8.6) that the value of M_v at dryout is

$$M_{vD} = H_{v0}\,[1 + H_{v0}^3]^{-\frac{1}{3}} \qquad (8.8)$$

and M_{vD} tends to unity if the initial dimensionless film thickness H_{v0} is sufficiently large. This result is shown as an asymptote in Fig. 5.

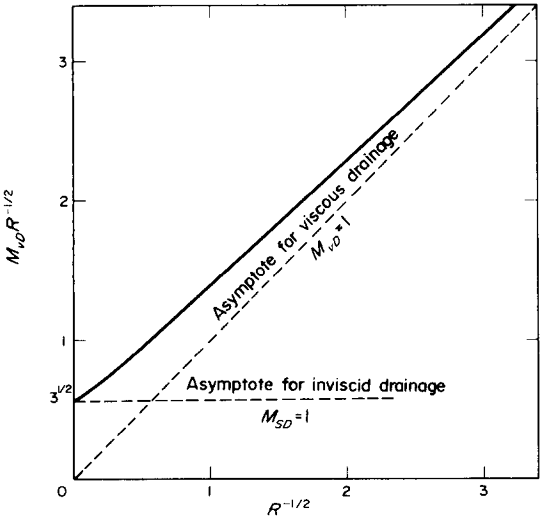

FIG. 5. Salt deposited at dryout of thick films.

Next we assume that drainage is such that the free-fall velocity obtains throughout the film. Equation (8.4) is reduced to the form

$$H_S(1 + H_S)\frac{\mathrm{d}c}{\mathrm{d}H_S} = -c \qquad (8.9)$$

by application of equation (6.3). Integration yields

$$\frac{c}{c_0} = \frac{H_{SO}}{H_S}\left[\frac{1 + H_S}{1 + H_{SO}}\right]. \qquad (8.10)$$

Define

$$M_S = \frac{c}{c_0} H_S = \frac{m}{c_0 L_S}. \qquad (8.11)$$

The value of M_S at dryout is

$$M_{SD} = \frac{H_{SO}}{1 + H_{SO}} \qquad (8.12)$$

which is unity when the original dimensionless film thickness is large. This result provides the other asymptote in Fig. 5, when it is noted from equations (6.5), (8.7) and (8.11) that

$$\frac{M_{vD}}{M_{SD}} = \frac{L_S}{L_v} = \left(\frac{R}{3}\right)^{\frac{1}{2}}. \qquad (8.13)$$

The accurate curve shown in Fig. 5 was derived numerically from equation (8.4) by the methods so far indicated in this section but changing the equation used to describe the thinning process as thinning progresses in the manner suggested in Sections 5 and 6. The relevant equations for increasing film thickness are (3.6), (4.6) and (6.2).

9. DEPOSITION OF SALT WITH A SMALL DIFFUSIVITY

When the diffusivity is small we still have the condition that concentration is invarient in the x-direction. Otherwise we shall employ the simplification that variations in the salt concentration only occur in the liquid layers immediately adjacent to the liquid–vapour interface, where the u-component of velocity can be assumed to

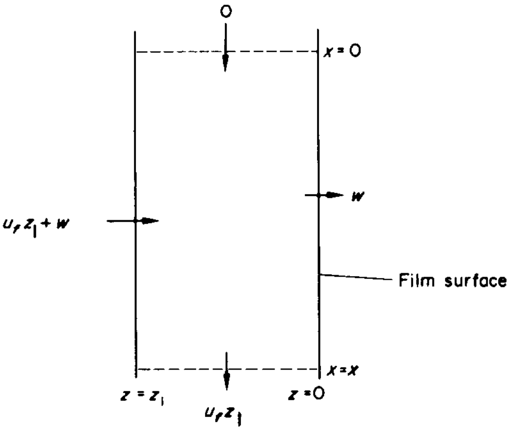

FIG. 6. System with slowly diffusing salt.

be uniform, whether the flow is purely viscous or not.

Let us define a quantity

$$m_1 = \int_0^\infty (c - c_0)\,\mathrm{d}z \qquad (9.1)$$

where z is measured normal to the interface into the liquid, as shown in Fig. 6. c_0 is the initial salt concentration which will obtain not far from the interface. Also note that the coordinate system is chosen such that the interface is stationary and thus the liquid has a velocity of $(-w)$ across it. We draw up the salt balance indicated in Fig. 6, choosing a distance z_1 from the interface such that $c \simeq c_0$. Noting that $u = 0$ at $x = 0$, the balance is

$$(u_f z_1 + wx)c_0 = u_f z_1 c_0 + u_f m_1 + x\frac{\mathrm{d}m_1}{\mathrm{d}t} \qquad (9.2)$$

where u_f is the value of u at the interface. Equation (9.2) reduces to

$$\frac{\mathrm{d}m_1}{\mathrm{d}t} + \frac{u_f m_1}{x} = wc_0. \qquad (9.3)$$

Equation (9.3) can be solved if u_f and $\mathrm{d}h/\mathrm{d}t$ can be expressed in terms of h. The appropriate expressions for purely viscous flow are obtained from Section 3 and then equation (9.3) becomes

$$(1 + H_v^3)\frac{\mathrm{d}M_{1v}}{\mathrm{d}H_v} - \frac{3}{2}H_v^2 M_{1v} + 1 = 0 \qquad (9.4)$$

after rearrangement. Integration gives

$$M_{1v} = (1 + H_v^3)^{\frac{1}{2}} \left[\int_0^{H_{v0}} \frac{\mathrm{d}H_v}{(1 + H_v^3)^{\frac{3}{2}}} - \int_0^{H_v} \frac{\mathrm{d}H_v}{(1 + H_v^3)^{\frac{3}{2}}}\right]. \qquad (9.5)$$

The total dimensionless mass of salt in the film is

$$M_v = M_{1v} + H_v. \qquad (9.6)$$

The salt deposited at dryout is obtained from equation (9.5) by putting $H_v = 0$ and, if the original film is thick enough, this quantity is

$$M_{vD} = \int_0^\infty \frac{\mathrm{d}H_v}{(1 + H_v^3)^{\frac{3}{2}}} = \frac{\Gamma(\frac{4}{3})\Gamma(\frac{7}{6})}{\Gamma(\frac{3}{2})} = 0{\cdot}9348 \qquad (9.7)$$

which is only about 6·5 per cent less than is obtained with a rapidly diffusing salt.

Let us next consider that the whole liquid film has the free fall velocity, as given by equation (4.1), and in consequence the relationship between time and film thickness is given by equation (6.3). Equation (9.3) reduces to

$$(1 + H_S)\frac{\mathrm{d}M_{1S}}{\mathrm{d}H_S} - M_{1S} + 1 = 0. \qquad (9.8)$$

Integration yields

$$M_S = M_{IS} + H_S = \frac{H_{SO} - H_S}{1 + H_{SO}} + H_S \qquad (9.9)$$

which is identical with the result obtained by equation (8.10) for the same situation but with a rapidly diffusing salt.

It is concluded that the diffusivity of the salt has little influence upon the amount of salt deposited at dryout.

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DRAINAGE ET EVAPORATION D'UN FILM LIQUIDE AVEC DEPOT DE SEL SUR UNE SURFACE CYLINDRIQUE HORIZONTALE

Résumé—On fait une analyse du drainage et de l'évaporation d'un film liquide sur une surface cylindrique horizontale qui forme la surface intérieure ou extérieure d'un tube. Les résultats représentés par la figure 2 donnent le temps de la première apparition de l'assèchement en fonction de l'épaisseur du film avec le nombre de Reynolds pour paramètre.

L'analyse du processus de drainage et d'évaporation est utilisée comme point de départ pour la détermination du dépôt de sel à l'asséchement et les résultats majeurs sont représentés par la Fig. 5. On remarque principalement que la quantité de sel déposé ne dépend pas substantiellement de la diffusivité moléculaire.

ABLEITUNG UND VERDAMPFUNG EINES FLÜSSIGKEITSFILMS MIT SALZNIEDERSCHLAG AN EINER HORIZONTALEN, ZYLINDRISCHEN OBERFLÄCHE

Zusammenfassung—Eine Analyse der Abströmung und Verdampfung eines Flüssigkeitsfilms an einer horizontalen, zylindrischen Oberfläche, welche die innere oder die äussere Oberfläche eines Rohres bildet, wurde durchgeführt. Die Ergebnisse für die Filmdicke sind in Fig. 2 als Funktion der Zeit für das erste Auftreten der Austrocknung, mit der Reynoldszahl als Parameter angegeben.

Die Analyse des Abströmungs- und Verdampfungsprozesses wurde als Ausgangspunkt für die Bestimmung des bei der Austrocknung abgelagerten Salzes benutzt. Die Hauptergebnisse sind in Fig. 5 dargestellt. Es ist bemerkenswert, dass keine deutliche Abhängigkeit für die Menge des abgelagerten Salzes vom molekularen Diffusionsvermögen vorhanden zu sein scheint.

ДРЕНАЖ И ИСПАРЕНИЕ ЖИДКОЙ ПЛЕНКИ С ОТЛОЖЕНИЕМ СОЛИ НА ГОРИЗОНТАЛЬНОЙ ЦИЛИНДРИЧЕСКОЙ ПОВЕРХНОСТИ

Аннотация—Анализируется дренаж и испарение жидкой пленки на горизонтальной цилиндрической поверхности, составляющей внутреннюю или внешнюю поверхность трубы. Результаты представлены на рис. 2 в виде зависимости времени начала дренажа от толщины пленки, а в качестве параметра взято число Рейнольдса. Анализ процесса дренажа и испарения используется как отправная точка при определении количества соли, отложенной при высыхании, и основные результаты представлены на рис. 5. Следует отметить отсутствие существенной зависимости между количеством отложившейся соли и коэффициентом молекулярной диффузии.